Maple 2018.2 Integration Test Results
on the problems in "7 Inverse hyperbolic functions/7.5 Inverse hyperbolic secant"

Test results for the 50 problems in "7.5.1 u (a+b arcsech(c x))^n.txt"

Problem 4: Unable to integrate problem.

$$\int x^4 \operatorname{arcsech}(a x)^3 \, dx$$

Optimal(type 4, 382 leaves, 14 steps):

$$-\frac{9 x \operatorname{arcsech}(a x)}{20 a^4} - \frac{x^3 \operatorname{arcsech}(a x)}{10 a^2} + \frac{x^5 \operatorname{arcsech}(a x)^3}{5} - \frac{9 \operatorname{arcsech}(a x)^2 \arctan\left(\frac{1}{a x} + \sqrt{\frac{1}{a x} - 1}\, \sqrt{\frac{1}{a x} + 1}\right)}{20 a^5} + \frac{\arctan\left(\frac{(a x + 1) \sqrt{\frac{-a x + 1}{a x + 1}}}{a x}\right)}{2 a^5}$$
$$+ \frac{9 I \operatorname{arcsech}(a x) \operatorname{polylog}\left(2, -I \left(\frac{1}{a x} + \sqrt{\frac{1}{a x} - 1}\, \sqrt{\frac{1}{a x} + 1}\right)\right)}{20 a^5} - \frac{9 I \operatorname{arcsech}(a x) \operatorname{polylog}\left(2, I \left(\frac{1}{a x} + \sqrt{\frac{1}{a x} - 1}\, \sqrt{\frac{1}{a x} + 1}\right)\right)}{20 a^5}$$
$$- \frac{9 I \operatorname{polylog}\left(3, -I \left(\frac{1}{a x} + \sqrt{\frac{1}{a x} - 1}\, \sqrt{\frac{1}{a x} + 1}\right)\right)}{20 a^5} + \frac{9 I \operatorname{polylog}\left(3, I \left(\frac{1}{a x} + \sqrt{\frac{1}{a x} - 1}\, \sqrt{\frac{1}{a x} + 1}\right)\right)}{20 a^5} + \frac{x (a x + 1) \sqrt{\frac{-a x + 1}{a x + 1}}}{20 a^4}$$
$$- \frac{9 x (a x + 1) \operatorname{arcsech}(a x)^2 \sqrt{\frac{-a x + 1}{a x + 1}}}{40 a^4} - \frac{3 x^3 (a x + 1) \operatorname{arcsech}(a x)^2 \sqrt{\frac{-a x + 1}{a x + 1}}}{20 a^2}$$

Result(type 8, 12 leaves):

$$\int x^4 \operatorname{arcsech}(a x)^3 \, dx$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int x (a + b \operatorname{arcsech}(c x))^2 \, dx$$

Optimal(type 3, 61 leaves, 4 steps):

$$\frac{x^2 (a + b \operatorname{arcsech}(c x))^2}{2} - \frac{b^2 \ln(x)}{c^2} - \frac{b (c x + 1) (a + b \operatorname{arcsech}(c x)) \sqrt{\frac{-c x + 1}{c x + 1}}}{c^2}$$

Result(type 3, 167 leaves):

$$\frac{a^2 x^2}{2} - \frac{b^2 \operatorname{arcsech}(c x)}{c^2} - \frac{b^2 \sqrt{\frac{c x + 1}{c x}}\, \sqrt{-\frac{c x - 1}{c x}}\, \operatorname{arcsech}(c x)\, x}{c} + \frac{b^2 x^2 \operatorname{arcsech}(c x)^2}{2} + \frac{b^2 \ln\left(\left(\frac{1}{c x} + \sqrt{-1 + \frac{1}{c x}}\, \sqrt{1 + \frac{1}{c x}}\right)^2 + 1\right)}{c^2}$$

$$+ a b \operatorname{arcsech}(c x) x^2 - \frac{a b \sqrt{-\frac{c x - 1}{c x}} \sqrt{\frac{c x + 1}{c x}} x}{c}$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{arcsech}(c x))^2}{x^5} \, dx$$

Optimal(type 3, 133 leaves, 5 steps):

$$-\frac{b^2}{32 x^4} - \frac{3 b^2 c^2}{32 x^2} + \frac{3 a b c^4 \operatorname{arcsech}(c x)}{16} + \frac{3 b^2 c^4 \operatorname{arcsech}(c x)^2}{32} - \frac{(a + b \operatorname{arcsech}(c x))^2}{4 x^4} + \frac{b (c x + 1) (a + b \operatorname{arcsech}(c x)) \sqrt{\frac{-c x + 1}{c x + 1}}}{8 x^4}$$
$$+ \frac{3 b c^2 (c x + 1) (a + b \operatorname{arcsech}(c x)) \sqrt{\frac{-c x + 1}{c x + 1}}}{16 x^2}$$

Result(type 3, 297 leaves):

$$c^4 \left(-\frac{a^2}{4 c^4 x^4} + b^2 \left(\frac{\operatorname{arcsech}(c x)^2 (c x - 1) (c x + 1)}{4 c^4 x^4} - \frac{\operatorname{arcsech}(c x)^2}{4 c^2 x^2} + \frac{\operatorname{arcsech}(c x) \sqrt{-\frac{c x - 1}{c x}} \sqrt{\frac{c x + 1}{c x}}}{8 c^3 x^3} + \frac{3 \operatorname{arcsech}(c x) \sqrt{-\frac{c x - 1}{c x}} \sqrt{\frac{c x + 1}{c x}}}{16 c x} \right.\right.$$
$$\left. + \frac{3 \operatorname{arcsech}(c x)^2}{32} + \frac{(c x - 1) (c x + 1)}{32 c^4 x^4} - \frac{1}{8 c^2 x^2} \right) + 2 a b \left(-\frac{\operatorname{arcsech}(c x)}{4 c^4 x^4} \right.$$
$$\left.\left. + \frac{\sqrt{-\frac{c x - 1}{c x}} \sqrt{\frac{c x + 1}{c x}} \left(3 \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2 x^2 + 1}} \right) c^4 x^4 + 3 c^2 x^2 \sqrt{-c^2 x^2 + 1} + 2 \sqrt{-c^2 x^2 + 1} \right)}{32 c^3 x^3 \sqrt{-c^2 x^2 + 1}} \right) \right)$$

Problem 15: Unable to integrate problem.

$$\int (a + b \operatorname{arcsech}(c x))^3 \, dx$$

Optimal(type 4, 257 leaves, 9 steps):

$$x (a + b \operatorname{arcsech}(c x))^3 - \frac{6 b (a + b \operatorname{arcsech}(c x))^2 \arctan\left(\frac{1}{c x} + \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}} \right)}{c}$$

$$+\frac{6 I b^2 (a+b \operatorname{arcsech}(c x)) \operatorname{polylog}\left(2, -I\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\sqrt{1+\frac{1}{c x}}\right)\right)}{c}$$

$$-\frac{6 I b^2 (a+b \operatorname{arcsech}(c x)) \operatorname{polylog}\left(2, I\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\sqrt{1+\frac{1}{c x}}\right)\right)}{c}-\frac{6 I b^3 \operatorname{polylog}\left(3, -I\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\sqrt{1+\frac{1}{c x}}\right)\right)}{c}$$

$$+\frac{6 I b^3 \operatorname{polylog}\left(3, I\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\sqrt{1+\frac{1}{c x}}\right)\right)}{c}$$

Result(type 8, 12 leaves):

$$\int (a+b \operatorname{arcsech}(c x))^3 \, dx$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \operatorname{arcsech}(c x))^3}{x^2} \, dx$$

Optimal(type 3, 98 leaves, 5 steps):

$$-\frac{6 b^2 (a+b \operatorname{arcsech}(c x))}{x}-\frac{(a+b \operatorname{arcsech}(c x))^3}{x}+\frac{6 b^3 (c x+1)\sqrt{\frac{-c x+1}{c x+1}}}{x}+\frac{3 b (c x+1) (a+b \operatorname{arcsech}(c x))^2\sqrt{\frac{-c x+1}{c x+1}}}{x}$$

Result(type 3, 226 leaves):

$$c\left(-\frac{a^3}{c x}+b^3\left(-\frac{\operatorname{arcsech}(c x)^3}{c x}+3 \operatorname{arcsech}(c x)^2\sqrt{-\frac{c x-1}{c x}}\sqrt{\frac{c x+1}{c x}}-\frac{6 \operatorname{arcsech}(c x)}{c x}+6\sqrt{-\frac{c x-1}{c x}}\sqrt{\frac{c x+1}{c x}}\right)+3 a b^2\left(-\frac{\operatorname{arcsech}(c x)^2}{c x}\right.\right.$$

$$\left.\left.+2 \operatorname{arcsech}(c x)\sqrt{-\frac{c x-1}{c x}}\sqrt{\frac{c x+1}{c x}}-\frac{2}{c x}\right)+3 a^2 b\left(-\frac{\operatorname{arcsech}(c x)}{c x}+\sqrt{-\frac{c x-1}{c x}}\sqrt{\frac{c x+1}{c x}}\right)\right)$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \operatorname{arcsech}(c x))^3}{x^4} \, dx$$

Optimal(type 3, 191 leaves, 8 steps):

$$\frac{2 b^3\left(\frac{-c x+1}{c x+1}\right)^{3/2}(c x+1)^3}{27 x^3}-\frac{2 b^2 (a+b \operatorname{arcsech}(c x))}{9 x^3}-\frac{4 b^2 c^2 (a+b \operatorname{arcsech}(c x))}{3 x}-\frac{(a+b \operatorname{arcsech}(c x))^3}{3 x^3}+\frac{14 b^3 c^2 (c x+1)\sqrt{\frac{-c x+1}{c x+1}}}{9 x}$$

$$+\frac{b (c x+1) (a+b \operatorname{arcsech}(c x))^2\sqrt{\frac{-c x+1}{c x+1}}}{3 x^3}+\frac{2 b c^2 (c x+1) (a+b \operatorname{arcsech}(c x))^2\sqrt{\frac{-c x+1}{c x+1}}}{3 x}$$

Result(type 3, 454 leaves):

$$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(\frac{\operatorname{arcsech}(cx)^3(cx-1)(cx+1)}{3c^3x^3}-\frac{\operatorname{arcsech}(cx)^3}{3cx}+\frac{\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{3c^2x^2}+\frac{2\operatorname{arcsech}(cx)^2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{3}\right.\right.$$
$$\left.+\frac{2\operatorname{arcsech}(cx)(cx-1)(cx+1)}{9c^3x^3}-\frac{14\operatorname{arcsech}(cx)}{9cx}+\frac{2\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{27c^2x^2}+\frac{40\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{27}\right)$$
$$+3ab^2\left(\frac{\operatorname{arcsech}(cx)^2(cx-1)(cx+1)}{3c^3x^3}-\frac{\operatorname{arcsech}(cx)^2}{3cx}+\frac{2\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{9c^2x^2}+\frac{4\operatorname{arcsech}(cx)\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{9}\right.$$
$$\left.\left.+\frac{2(cx-1)(cx+1)}{27c^3x^3}-\frac{14}{27cx}\right)+3a^2b\left(-\frac{\operatorname{arcsech}(cx)}{3c^3x^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}(2c^2x^2+1)}{9c^2x^2}\right)\right)$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x^2(a+b\operatorname{arcsech}(cx))^3}\,dx$$

Optimal(type 4, 104 leaves, 6 steps):

$$\frac{1}{2b^2x(a+b\operatorname{arcsech}(cx))}-\frac{c\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\operatorname{arcsech}(cx)\right)}{2b^3}+\frac{c\operatorname{Chi}\left(\frac{a}{b}+\operatorname{arcsech}(cx)\right)\sinh\left(\frac{a}{b}\right)}{2b^3}+\frac{(cx+1)\sqrt{\frac{-cx+1}{cx+1}}}{2bx(a+b\operatorname{arcsech}(cx))^2}$$

Result(type 4, 243 leaves):

$$c\left(-\frac{\left(\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx-1\right)(b\operatorname{arcsech}(cx)+a-b)}{4cxb^2\left(\operatorname{arcsech}(cx)^2b^2+2\operatorname{arcsech}(cx)ab+a^2\right)}-\frac{e^{\frac{a}{b}}\operatorname{Ei}_1\left(\frac{a}{b}+\operatorname{arcsech}(cx)\right)}{4b^3}+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx+1}{4bcx(a+b\operatorname{arcsech}(cx))^2}\right.$$
$$\left.+\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\,cx+1}{4b^2cx(a+b\operatorname{arcsech}(cx))}+\frac{e^{-\frac{a}{b}}\operatorname{Ei}_1\left(-\operatorname{arcsech}(cx)-\frac{a}{b}\right)}{4b^3}\right)$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int\frac{a+b\operatorname{arcsech}(cx)}{(ex+d)^3}\,dx$$

Optimal(type 3, 267 leaves, 11 steps):

$$\frac{-a-b\,\mathrm{arcsech}(cx)}{2e\,(ex+d)^2}+\frac{b\,c^2\arctan\left(\frac{c^2xd+e}{\sqrt{c^2d^2-e^2}\,\sqrt{-c^2x^2+1}}\right)\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}}{2\,(c^2d^2-e^2)^{3/2}}+\frac{b\,\mathrm{arctanh}\left(\sqrt{-c^2x^2+1}\right)\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}}{2d^2e}$$

$$+\frac{b\arctan\left(\frac{c^2xd+e}{\sqrt{c^2d^2-e^2}\,\sqrt{-c^2x^2+1}}\right)\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}}{2d^2\sqrt{c^2d^2-e^2}}+\frac{b\,e\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\sqrt{-c^2x^2+1}}{2d\,(c^2d^2-e^2)\,(ex+d)}$$

Result(type 3, 1089 leaves):

$$-\frac{c^2a}{2\,(cex+cd)^2e}-\frac{c^2b\,\mathrm{arcsech}(cx)}{2\,(cex+cd)^2e}+\frac{c^4b\sqrt{-\frac{cx-1}{cx}}\,x^2\sqrt{\frac{cx+1}{cx}}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{2\sqrt{-c^2x^2+1}\,(cd-e)\,(cd+e)\,(cex+cd)}$$

$$+\frac{c^4b\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,d\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{2e\sqrt{-c^2x^2+1}\,(cd-e)\,(cd+e)\,(cex+cd)}-\frac{c^4b\sqrt{-\frac{cx-1}{cx}}\,x^2\sqrt{\frac{cx+1}{cx}}\,\ln\left(\frac{2\left(\sqrt{-c^2x^2+1}\,\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,e+c^2xd+e\right)}{cex+cd}\right)}{\sqrt{-c^2x^2+1}\,(cd-e)\,(cd+e)\,(cex+cd)\sqrt{-\frac{c^2d^2-e^2}{e^2}}}$$

$$-\frac{c^4b\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,d\ln\left(\frac{2\left(\sqrt{-c^2x^2+1}\,\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,e+c^2xd+e\right)}{cex+cd}\right)}{e\sqrt{-c^2x^2+1}\,(cd-e)\,(cd+e)\,(cex+cd)\sqrt{-\frac{c^2d^2-e^2}{e^2}}}+\frac{c^2be\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}}{2\,(cd-e)\,(cd+e)\,d\,(cex+cd)}$$

$$-\frac{c^2be^2\sqrt{-\frac{cx-1}{cx}}\,x^2\sqrt{\frac{cx+1}{cx}}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{2\sqrt{-c^2x^2+1}\,(cd-e)\,(cd+e)\,d^2\,(cex+cd)}-\frac{c^2be\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{2\sqrt{-c^2x^2+1}\,(cd-e)\,(cd+e)\,d\,(cex+cd)}$$

$$+\frac{c^2be^2\sqrt{-\frac{cx-1}{cx}}\,x^2\sqrt{\frac{cx+1}{cx}}\,\ln\left(\frac{2\left(\sqrt{-c^2x^2+1}\,\sqrt{-\frac{c^2d^2-e^2}{e^2}}\,e+c^2xd+e\right)}{cex+cd}\right)}{2\sqrt{-c^2x^2+1}\,(cd-e)\,(cd+e)\,d^2\,(cex+cd)\sqrt{-\frac{c^2d^2-e^2}{e^2}}}$$

$$+\frac{c^2 b e \sqrt{-\frac{cx-1}{cx}}\, x \sqrt{\frac{cx+1}{cx}} \ln\left(\frac{2\left(\sqrt{-c^2x^2+1}\sqrt{-\frac{c^2d^2-e^2}{e^2}}\, e+c^2xd+e\right)}{cex+cd}\right)}{2\sqrt{-c^2x^2+1}\,(cd-e)\,(cd+e)\,d\,(cex+cd)\sqrt{-\frac{c^2d^2-e^2}{e^2}}}$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int (ex+d)^{3/2}\,(a+b\,\mathrm{arcsech}(cx))\,dx$$

Optimal(type 4, 302 leaves, 21 steps):

$$\frac{2(ex+d)^{5/2}(a+b\,\mathrm{arcsech}(cx))}{5e}-\frac{28bd\,\mathrm{EllipticE}\left(\frac{\sqrt{-cx+1}\sqrt{2}}{2},\sqrt{2}\sqrt{\frac{e}{cd+e}}\right)\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{ex+d}}{15c\sqrt{\frac{c(ex+d)}{cd+e}}}$$

$$-\frac{4b\left(2c^2d^2+e^2\right)\mathrm{EllipticF}\left(\frac{\sqrt{-cx+1}\sqrt{2}}{2},\sqrt{2}\sqrt{\frac{e}{cd+e}}\right)\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(ex+d)}{cd+e}}}{15c^3\sqrt{ex+d}}$$

$$-\frac{4bd^3\,\mathrm{EllipticPi}\left(\frac{\sqrt{-cx+1}\sqrt{2}}{2},2,\sqrt{2}\sqrt{\frac{e}{cd+e}}\right)\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(ex+d)}{cd+e}}}{5e\sqrt{ex+d}}-\frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{ex+d}\sqrt{-c^2x^2+1}}{15c^2}$$

Result(type 4, 829 leaves):

$$\frac{1}{e}\left(2\left(\frac{(ex+d)^{5/2}a}{5}+b\left(\frac{(ex+d)^{5/2}\,\mathrm{arcsech}(cx)}{5}\right.\right.\right.$$

$$-\frac{1}{15c\sqrt{\frac{c}{cd+e}}\left((ex+d)^2c^2-2(ex+d)c^2d+c^2d^2-e^2\right)}\left(2e^2\sqrt{-\frac{c(ex+d)-cd-e}{cxe}}\,x\sqrt{\frac{c(ex+d)-cd+e}{cxe}}\left(\sqrt{\frac{c}{cd+e}}\,(ex+d)^{5/2}c^2\right.\right.$$

$$+9\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)c^2d^2$$

$$-7\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\,\mathrm{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)c^2d^2$$

$$-3\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\,\mathrm{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\frac{cd+e}{cd},\frac{\sqrt{\frac{c}{cd-e}}}{\sqrt{\frac{c}{cd+e}}}\right)c^2d^2-2\sqrt{\frac{c}{cd+e}}\,(ex+d)^{3/2}c^2d$$

$$-7\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)cde$$

$$+7\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\,\mathrm{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)cde+\sqrt{\frac{c}{cd+e}}\sqrt{ex+d}\,c^2d^2$$

$$+\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd+e}},\sqrt{\frac{cd+e}{cd-e}}\right)e^2-\sqrt{\frac{c}{cd+e}}\sqrt{ex+d}\,e^2\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)$$

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Problem 31: Result is not expressed in closed-form.
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$$\int\frac{a+b\,\mathrm{arcsech}(cx)}{x(ex^2+d)}\,dx$$

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Optimal(type 4, 551 leaves, 19 steps):
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$$\frac{(a+b\,\mathrm{arcsech}(cx))^2}{2bd}-\frac{(a+b\,\mathrm{arcsech}(cx))\ln\left(1-\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$

$$-\frac{(a+b\,\mathrm{arcsech}(cx))\ln\left(1+\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}$$

$$-\frac{(a+b\,\mathrm{arcsech}(c x))\ln\left(1-\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)}{2 d}$$

$$-\frac{(a+b\,\mathrm{arcsech}(c x))\ln\left(1+\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)}{2 d}-\frac{b\,\mathrm{polylog}\left(2,-\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{2 d}$$

$$-\frac{b\,\mathrm{polylog}\left(2,\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{2 d}-\frac{b\,\mathrm{polylog}\left(2,-\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)}{2 d}$$

$$-\frac{b\,\mathrm{polylog}\left(2,\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)}{2 d}$$

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Result(type 7, 487 leaves):
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$$\frac{a\ln(c x)}{d}-\frac{a\ln\left(c^2 e x^2+c^2 d\right)}{2 d}+\frac{b\,\mathrm{arcsech}(c x)^2}{2 d}-\frac{1}{2 d}\Bigg(b\Bigg($$

$$\sum_{\_R1=RootOf\left(c^2 d\,\_Z^4+(2 c^2 d+4 e)\,\_Z^2+c^2 d\right)}\frac{\left(\_R1^2 c^2 d+2 c^2 d+4 e\right)\left(\mathrm{arcsech}(c x)\ln\left(\frac{\_R1-\frac{1}{c x}-\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-\frac{1}{c x}-\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}}{\_R1}\right)\right)}{\_R1^2 c^2 d+c^2 d+2 e}\Bigg)\Bigg)$$

$$+\frac{1}{2}\Bigg(b c^2\Bigg($$

$$\sum_{\_R1=RootOf(c^2 d \_Z^4+(2 c^2 d+4 e) \_Z^2+c^2 d)} \frac{\operatorname{arcsech}(c x) \ln\left(\frac{\_R1-\frac{1}{c x}-\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-\frac{1}{c x}-\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}}{\_R1}\right)}{\_R1^2 c^2 d+c^2 d+2 e}\Bigg)\Bigg)$$

$$+\frac{1}{d}\Bigg(b e\Bigg(\sum_{\_R1=RootOf(c^2 d \_Z^4+(2 c^2 d+4 e) \_Z^2+c^2 d)} \frac{\operatorname{arcsech}(c x) \ln\left(\frac{\_R1-\frac{1}{c x}-\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-\frac{1}{c x}-\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}}{\_R1}\right)}{\_R1^2 c^2 d+c^2 d+2 e}\Bigg)\Bigg)$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{x\,(a+b\operatorname{arcsech}(c x))}{(e x^2+d)^2}\,dx$$

Optimal(type 3, 124 leaves, 8 steps):

$$\frac{-a-b\operatorname{arcsech}(c x)}{2 e\,(e x^2+d)}+\frac{b\operatorname{arctanh}\left(\sqrt{-c^2 x^2+1}\right)\sqrt{\frac{1}{c x+1}}\,\sqrt{c x+1}}{2 d e}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{e}\,\sqrt{-c^2 x^2+1}}{\sqrt{c^2 d+e}}\right)\sqrt{\frac{1}{c x+1}}\,\sqrt{c x+1}}{2 d\sqrt{e}\,\sqrt{c^2 d+e}}$$

Result(type 3, 839 leaves):

$$-\frac{c^2 a}{2 e\,(c^2 e x^2+c^2 d)}-\frac{c^2 b\operatorname{arcsech}(c x)}{2 e\,(c^2 e x^2+c^2 d)}-\frac{c^3 b\sqrt{-\frac{c x-1}{c x}}\,x\sqrt{\frac{c x+1}{c x}}\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2 x^2+1}}\right)}{2\sqrt{-c^2 x^2+1}\,\left(e+\sqrt{-c^2 d e}\right)\left(-e+\sqrt{-c^2 d e}\right)}$$

$$+\frac{c^3 b\sqrt{-\frac{c x-1}{c x}}\,x\sqrt{\frac{c x+1}{c x}}\ln\left(\frac{2\left(\sqrt{-c^2 x^2+1}\,\sqrt{\frac{c^2 d+e}{e}}\,e+\sqrt{-c^2 d e}\,c x+e\right)}{c e x+\sqrt{-c^2 d e}}\right)}{4\sqrt{-c^2 x^2+1}\,\left(e+\sqrt{-c^2 d e}\right)\left(-e+\sqrt{-c^2 d e}\right)\sqrt{\frac{c^2 d+e}{e}}}$$

$$+\frac{c^3 b \sqrt{-\frac{cx-1}{cx}}\, x \sqrt{\frac{cx+1}{cx}} \ln\left(-\frac{2\left(\sqrt{-c^2x^2+1}\,\sqrt{\frac{c^2d+e}{e}}\, e-\sqrt{-c^2de}\, cx+e\right)}{-cex+\sqrt{-c^2de}}\right)}{4\sqrt{-c^2x^2+1}\,\left(e+\sqrt{-c^2de}\right)\left(-e+\sqrt{-c^2de}\right)\sqrt{\frac{c^2d+e}{e}}}-\frac{cb\sqrt{-\frac{cx-1}{cx}}\, x\sqrt{\frac{cx+1}{cx}}\, \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right) e}{2\sqrt{-c^2x^2+1}\, d\left(e+\sqrt{-c^2de}\right)\left(-e+\sqrt{-c^2de}\right)}$$

$$+\frac{cb\sqrt{-\frac{cx-1}{cx}}\, x\sqrt{\frac{cx+1}{cx}}\, e\ln\left(\frac{2\left(\sqrt{-c^2x^2+1}\,\sqrt{\frac{c^2d+e}{e}}\, e+\sqrt{-c^2de}\, cx+e\right)}{cex+\sqrt{-c^2de}}\right)}{4\sqrt{-c^2x^2+1}\, d\left(e+\sqrt{-c^2de}\right)\left(-e+\sqrt{-c^2de}\right)\sqrt{\frac{c^2d+e}{e}}}$$

$$+\frac{cb\sqrt{-\frac{cx-1}{cx}}\, x\sqrt{\frac{cx+1}{cx}}\, e\ln\left(-\frac{2\left(\sqrt{-c^2x^2+1}\,\sqrt{\frac{c^2d+e}{e}}\, e-\sqrt{-c^2de}\, cx+e\right)}{-cex+\sqrt{-c^2de}}\right)}{4\sqrt{-c^2x^2+1}\, d\left(e+\sqrt{-c^2de}\right)\left(-e+\sqrt{-c^2de}\right)\sqrt{\frac{c^2d+e}{e}}}$$

Problem 33: Result is not expressed in closed-form.

$$\int \frac{a+b\operatorname{arcsech}(cx)}{x\left(ex^2+d\right)^2}\, dx$$

Optimal(type 4, 656 leaves, 25 steps):

$$-\frac{e\left(a+b\operatorname{arcsech}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b\operatorname{arcsech}(cx)\right)^2}{2bd^2}-\frac{\left(a+b\operatorname{arcsech}(cx)\right)\ln\left(1-\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$

$$-\frac{\left(a+b\operatorname{arcsech}(cx)\right)\ln\left(1+\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$

$$-\frac{\left(a+b\operatorname{arcsech}(cx)\right)\ln\left(1-\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

$$-\frac{(a+b\,\mathrm{arcsech}(cx))\ln\left(1+\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}-\frac{b\,\mathrm{polylog}\left(2,-\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}$$

$$-\frac{b\,\mathrm{polylog}\left(2,\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{b\,\mathrm{polylog}\left(2,-\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}$$

$$-\frac{b\,\mathrm{polylog}\left(2,\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}+\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{c^2d+e}}{cx\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}}\right)\sqrt{e}\sqrt{-1+\frac{1}{c^2x^2}}}{2d^2\sqrt{c^2d+e}\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}$$

Result(type 7, 638 leaves):

$$\frac{a c^2}{2d\left(c^2ex^2+c^2d\right)}-\frac{a\ln\left(c^2ex^2+c^2d\right)}{2d^2}+\frac{a\ln(cx)}{d^2}+\frac{b\,\mathrm{arcsech}(cx)^2}{2d^2}-\frac{b c^2x^2\,\mathrm{arcsech}(cx)\,e}{2\left(c^2ex^2+c^2d\right)d^2}$$

$$-\frac{b\sqrt{\left(c^2d+e\right)e}\,\mathrm{arctanh}\left(\frac{2c^2d\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2+2c^2d+4e}{4\sqrt{c^2de+e^2}}\right)}{2d^2\left(c^2d+e\right)}-\frac{1}{2d^2}\Bigg(b\Bigg($$

$$\sum_{\_R1=RootOf\left(c^2d\,\_Z^4+\left(2c^2d+4e\right)\_Z^2+c^2d\right)}\frac{\left(\_R1^2c^2d+2c^2d+4e\right)\left(\mathrm{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{\_R1}\right)\right)}{\_R1^2c^2d+c^2d+2e}\Bigg)\Bigg)$$

$$+\frac{1}{2d}\Bigg(bc^2\Bigg($$

$$\sum_{\_R1=RootOf(c^2 d\_Z^4+(2c^2 d+4e)\_Z^2+c^2 d)} \frac{\operatorname{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{\_R1}\right)}{\_R1^2 c^2 d+c^2 d+2e}\Bigg)\Bigg)$$

$$+\frac{1}{d^2}\Bigg(be\Bigg(\sum_{\_R1=RootOf(c^2 d\_Z^4+(2c^2 d+4e)\_Z^2+c^2 d)} \frac{\operatorname{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{\_R1}\right)}{\_R1^2 c^2 d+c^2 d+2e}\Bigg)\Bigg)$$

Problem 34: Result is not expressed in closed-form.

$$\int \frac{x^4 (a+b\operatorname{arcsech}(cx))}{(ex^2+d)^2}\,dx$$

Optimal(type 4, 868 leaves, 50 steps):

$$\frac{x(a+b\operatorname{arcsech}(cx))}{e^2}-\frac{b\arctan\left(\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)}{ce^2}+\frac{3(a+b\operatorname{arcsech}(cx))\ln\left(1-\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)\sqrt{-d}}{4e^{5/2}}$$

$$-\frac{3(a+b\operatorname{arcsech}(cx))\ln\left(1+\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)\sqrt{-d}}{4e^{5/2}}$$

$$+\frac{3(a+b\operatorname{arcsech}(cx))\ln\left(1-\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)\sqrt{-d}}{4e^{5/2}}$$

$$-\frac{3(a+b\operatorname{arcsech}(cx))\ln\left(1+\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)\sqrt{-d}}{4e^{5/2}}$$

$$-\frac{3 b \operatorname{polylog}\left(2, -\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)\sqrt{-d}}{4 e^{5/2}}+\frac{3 b \operatorname{polylog}\left(2, \frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)\sqrt{-d}}{4 e^{5/2}}$$

$$-\frac{3 b \operatorname{polylog}\left(2, -\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)\sqrt{-d}}{4 e^{5/2}}+\frac{3 b \operatorname{polylog}\left(2, \frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)\sqrt{-d}}{4 e^{5/2}}$$

$$-\frac{d\,(a+b\operatorname{arcsech}(cx))}{4 e^2\left(-\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)}+\frac{d\,(a+b\operatorname{arcsech}(cx))}{4 e^2\left(\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)}+\frac{b\,d\arctan\left(\frac{\sqrt{1+\frac{1}{cx}}\,\sqrt{cd-\sqrt{-d}\,\sqrt{e}}}{\sqrt{-1+\frac{1}{cx}}\,\sqrt{cd+\sqrt{-d}\,\sqrt{e}}}\right)}{2 e^2\sqrt{cd-\sqrt{-d}\,\sqrt{e}}\,\sqrt{cd+\sqrt{-d}\,\sqrt{e}}}+\frac{b\,d\arctan\left(\frac{\sqrt{1+\frac{1}{cx}}\,\sqrt{cd+\sqrt{-d}\,\sqrt{e}}}{\sqrt{-1+\frac{1}{cx}}\,\sqrt{cd-\sqrt{-d}\,\sqrt{e}}}\right)}{2 e^2\sqrt{cd-\sqrt{-d}\,\sqrt{e}}\,\sqrt{cd+\sqrt{-d}\,\sqrt{e}}}$$

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Result(type ?, 2015 leaves): Display of huge result suppressed!
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Problem 35: Result is not expressed in closed-form.
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$$\int \frac{x^2\,(a+b\operatorname{arcsech}(cx))}{(e x^2+d)^2}\,dx$$

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Optimal(type 4, 821 leaves, 27 steps):
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$$\frac{(a+b\operatorname{arcsech}(cx))\ln\left(1-\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{4 e^{3/2}\sqrt{-d}}-\frac{(a+b\operatorname{arcsech}(cx))\ln\left(1+\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{4 e^{3/2}\sqrt{-d}}$$

$$+\frac{(a+b\operatorname{arcsech}(cx))\ln\left(1-\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)}{4 e^{3/2}\sqrt{-d}}$$

$$-\frac{(a+b\operatorname{arcsech}(cx))\ln\left(1+\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)}{4 e^{3/2}\sqrt{-d}}-\frac{b \operatorname{polylog}\left(2, -\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{4 e^{3/2}\sqrt{-d}}$$

$$+\frac{b\,\mathrm{polylog}\left(2, \frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{4 e^{3/2}\sqrt{-d}} - \frac{b\,\mathrm{polylog}\left(2, -\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)}{4 e^{3/2}\sqrt{-d}}$$

$$+\frac{b\,\mathrm{polylog}\left(2, \frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)}{4 e^{3/2}\sqrt{-d}} + \frac{a+b\,\mathrm{arcsech}(cx)}{4e\left(-\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)} + \frac{-a-b\,\mathrm{arcsech}(cx)}{4e\left(\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)}$$

$$-\frac{b\arctan\left(\frac{\sqrt{1+\frac{1}{cx}}\,\sqrt{cd-\sqrt{-d}\,\sqrt{e}}}{\sqrt{-1+\frac{1}{cx}}\,\sqrt{cd+\sqrt{-d}\,\sqrt{e}}}\right)}{2e\sqrt{cd-\sqrt{-d}\,\sqrt{e}}\,\sqrt{cd+\sqrt{-d}\,\sqrt{e}}} - \frac{b\arctan\left(\frac{\sqrt{1+\frac{1}{cx}}\,\sqrt{cd+\sqrt{-d}\,\sqrt{e}}}{\sqrt{-1+\frac{1}{cx}}\,\sqrt{cd-\sqrt{-d}\,\sqrt{e}}}\right)}{2e\sqrt{cd-\sqrt{-d}\,\sqrt{e}}\,\sqrt{cd+\sqrt{-d}\,\sqrt{e}}}$$

Result(type 7, 1879 leaves):

$$-\frac{c^2 a x}{2e\left(c^2 e x^2+c^2 d\right)} + \frac{a\arctan\left(\frac{xe}{\sqrt{de}}\right)}{2e\sqrt{de}} - \frac{c^2 b\,\mathrm{arcsech}(cx)\,x}{2e\left(c^2 e x^2+c^2 d\right)} - \frac{1}{4e}\left(cb\left(\sum_{\_R1=RootOf\left(c^2 d\,\_Z^4+\left(2c^2 d+4e\right)\_Z^2+c^2 d\right)} \frac{\_R1\left(\mathrm{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{\_R1}\right)\right)}{\_R1^2 c^2 d+c^2 d+2e}\right.\right.$$

$$\left.\left.+\frac{b\sqrt{\left(c^2 d+2\sqrt{\left(c^2 d+e\right)e}+2e\right)d}\,\arctan\left(\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)d}{\sqrt{\left(c^2 d+2\sqrt{\left(c^2 d+e\right)e}+2e\right)d}}\right)}{2c^2 e d^2}\right.\right.$$

$$- \frac{b \sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d} \arctan\left(\frac{c \left(\frac{1}{c x} + \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}\right) d}{\sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d}}\right) \sqrt{(c^2 d + e) e}}{c^4 e d^3}$$

$$+ \frac{b \sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d} \arctan\left(\frac{c \left(\frac{1}{c x} + \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}\right) d}{\sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d}}\right)}{c^4 d^3}$$

$$+ \frac{b \sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d} \arctan\left(\frac{c \left(\frac{1}{c x} + \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}\right) d}{\sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d}}\right) \sqrt{(c^2 d + e) e}}{2 c^2 e (c^2 d + e) d^2}$$

$$- \frac{b \sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d} \arctan\left(\frac{c \left(\frac{1}{c x} + \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}\right) d}{\sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d}}\right)}{c^2 (c^2 d + e) d^2}$$

$$+ \frac{b \sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d} \arctan\left(\frac{c \left(\frac{1}{c x} + \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}\right) d}{\sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d}}\right) \sqrt{(c^2 d + e) e}}{c^4 (c^2 d + e) d^3}$$

$$- \frac{b \sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d} \arctan\left(\frac{c \left(\frac{1}{c x} + \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}\right) d}{\sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d}}\right) e}{c^4 (c^2 d + e) d^3}$$

$$+ \frac{b \sqrt{-\left(c^2 d - 2 \sqrt{(c^2 d + e) e} + 2 e\right) d} \operatorname{arctanh}\left(\frac{c \left(\frac{1}{c x} + \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}\right) d}{\sqrt{\left(-c^2 d + 2 \sqrt{(c^2 d + e) e} - 2 e\right) d}}\right)}{2 c^2 e d^2}$$

$$+ \frac{b \sqrt{-\left(c^2 d - 2 \sqrt{(c^2 d + e) e} + 2 e\right) d} \operatorname{arctanh}\left(\frac{c \left(\frac{1}{c x} + \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}\right) d}{\sqrt{\left(-c^2 d + 2 \sqrt{(c^2 d + e) e} - 2 e\right) d}}\right) \sqrt{(c^2 d + e) e}}{c^4 e d^3}$$

$$+\frac{b\sqrt{-\left(c^2 d-2\sqrt{(c^2 d+e) e}+2 e\right) d}\,\operatorname{arctanh}\left(\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right) d}{\sqrt{\left(-c^2 d+2\sqrt{(c^2 d+e) e}-2 e\right) d}}\right)}{c^4 d^3}$$

$$-\frac{b\sqrt{-\left(c^2 d-2\sqrt{(c^2 d+e) e}+2 e\right) d}\,\operatorname{arctanh}\left(\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right) d}{\sqrt{\left(-c^2 d+2\sqrt{(c^2 d+e) e}-2 e\right) d}}\right)\sqrt{(c^2 d+e) e}}{2 c^2 e (c^2 d+e) d^2}$$

$$-\frac{b\sqrt{-\left(c^2 d-2\sqrt{(c^2 d+e) e}+2 e\right) d}\,\operatorname{arctanh}\left(\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right) d}{\sqrt{\left(-c^2 d+2\sqrt{(c^2 d+e) e}-2 e\right) d}}\right)}{c^2 (c^2 d+e) d^2}$$

$$-\frac{b\sqrt{-\left(c^2 d-2\sqrt{(c^2 d+e) e}+2 e\right) d}\,\operatorname{arctanh}\left(\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right) d}{\sqrt{\left(-c^2 d+2\sqrt{(c^2 d+e) e}-2 e\right) d}}\right)\sqrt{(c^2 d+e) e}}{c^4 (c^2 d+e) d^3}$$

$$-\frac{b\sqrt{-\left(c^2 d-2\sqrt{(c^2 d+e) e}+2 e\right) d}\,\operatorname{arctanh}\left(\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right) d}{\sqrt{\left(-c^2 d+2\sqrt{(c^2 d+e) e}-2 e\right) d}}\right) e}{c^4 (c^2 d+e) d^3}$$

$$+\frac{1}{4 e}\left(c b\left(\sum_{\_R1=RootOf(c^2 d\,\_Z^4+(2 c^2 d+4 e)\,\_Z^2+c^2 d)}\frac{\operatorname{arcsech}(c x)\ln\left(\frac{\_R1-\frac{1}{c x}-\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-\frac{1}{c x}-\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}}{\_R1}\right)}{\_R1\left(\_R1^2 c^2 d+c^2 d+2 e\right)}\right)\right)$$

Problem 36: Result is not expressed in closed-form.

$$\int\frac{a+b\operatorname{arcsech}(c x)}{x^2\left(e x^2+d\right)^2}\,dx$$

Optimal(type 4, 872 leaves, 50 steps):

$$-\frac{a}{d^2 x}-\frac{b\,\mathrm{arcsech}(c x)}{d^2 x}-\frac{3\,(a+b\,\mathrm{arcsech}(c x))\ln\left(1-\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)\sqrt{e}}{4\,(-d)^{5/2}}$$

$$+\frac{3\,(a+b\,\mathrm{arcsech}(c x))\ln\left(1+\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)\sqrt{e}}{4\,(-d)^{5/2}}$$

$$-\frac{3\,(a+b\,\mathrm{arcsech}(c x))\ln\left(1-\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)\sqrt{e}}{4\,(-d)^{5/2}}$$

$$+\frac{3\,(a+b\,\mathrm{arcsech}(c x))\ln\left(1+\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)\sqrt{e}}{4\,(-d)^{5/2}}+\frac{3\,b\,\mathrm{polylog}\left(2,-\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)\sqrt{e}}{4\,(-d)^{5/2}}$$

$$-\frac{3\,b\,\mathrm{polylog}\left(2,\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)\sqrt{e}}{4\,(-d)^{5/2}}+\frac{3\,b\,\mathrm{polylog}\left(2,-\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)\sqrt{e}}{4\,(-d)^{5/2}}$$

$$-\frac{3\,b\,\mathrm{polylog}\left(2,\frac{c\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)\sqrt{e}}{4\,(-d)^{5/2}}+\frac{e\,(a+b\,\mathrm{arcsech}(c x))}{4\,d^2\left(-\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)}-\frac{e\,(a+b\,\mathrm{arcsech}(c x))}{4\,d^2\left(\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)}+\frac{b\,c\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}}{d^2}$$

$$-\frac{b\,e\arctan\left(\frac{\sqrt{1+\frac{1}{c x}}\,\sqrt{c d-\sqrt{-d}\,\sqrt{e}}}{\sqrt{-1+\frac{1}{c x}}\,\sqrt{c d+\sqrt{-d}\,\sqrt{e}}}\right)}{2\,d^2\sqrt{c d-\sqrt{-d}\,\sqrt{e}}\,\sqrt{c d+\sqrt{-d}\,\sqrt{e}}}-\frac{b\,e\arctan\left(\frac{\sqrt{1+\frac{1}{c x}}\,\sqrt{c d+\sqrt{-d}\,\sqrt{e}}}{\sqrt{-1+\frac{1}{c x}}\,\sqrt{c d-\sqrt{-d}\,\sqrt{e}}}\right)}{2\,d^2\sqrt{c d-\sqrt{-d}\,\sqrt{e}}\,\sqrt{c d+\sqrt{-d}\,\sqrt{e}}}$$

Result(type 7, 1951 leaves):

$$- \frac{a}{d^2 x} - \frac{a e c^2 x}{2 d^2 (c^2 e x^2 + c^2 d)} - \frac{3 a e \arctan\left(\frac{x e}{\sqrt{d e}}\right)}{2 d^2 \sqrt{d e}} + \frac{1}{4 d^2} \left(3 c b e \left(\sum_{\_R1 = RootOf(c^2 d \_Z^4 + (2 c^2 d + 4 e) \_Z^2 + c^2 d)} \frac{\_R1 \left(\operatorname{arcsech}(c x) \ln\left(\frac{\_R1 - \frac{1}{c x} - \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{\_R1} \right) + \operatorname{dilog}\left(\frac{\_R1 - \frac{1}{c x} - \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{\_R1} \right) \right)}{\_R1^2 c^2 d + c^2 d + 2 e} \right) \right)$$

$$- \frac{1}{4 d^2} \left(3 c b e \left(\sum_{\_R1 = RootOf(c^2 d \_Z^4 + (2 c^2 d + 4 e) \_Z^2 + c^2 d)} \frac{\operatorname{arcsech}(c x) \ln\left(\frac{\_R1 - \frac{1}{c x} - \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{\_R1} \right) + \operatorname{dilog}\left(\frac{\_R1 - \frac{1}{c x} - \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{\_R1} \right)}{\_R1 (\_R1^2 c^2 d + c^2 d + 2 e)} \right) \right)$$

$$- \frac{b \operatorname{arcsech}(c x)}{d^2 x} - \frac{b \sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d}\; e \arctan\left(\frac{c \left(\frac{1}{c x} + \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}} \right) d}{\sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d}} \right) \sqrt{(c^2 d + e) e}}{c^4 d^5}$$

$$- \frac{b \sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d}\; e^2 \arctan\left(\frac{c \left(\frac{1}{c x} + \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}} \right) d}{\sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d}} \right)}{c^2 d^4 (c^2 d + e)}$$

$$- \frac{b \sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d}\; e^3 \arctan\left(\frac{c \left(\frac{1}{c x} + \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}} \right) d}{\sqrt{\left(c^2 d + 2 \sqrt{(c^2 d + e) e} + 2 e\right) d}} \right)}{c^4 d^5 (c^2 d + e)}$$

$$+\frac{b\sqrt{-\left(c^2 d-2\sqrt{(c^2 d+e)e}+2e\right)d}\; e \operatorname{arctanh}\left(\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)d}{\sqrt{\left(-c^2 d+2\sqrt{(c^2 d+e)e}-2e\right)d}}\right)\sqrt{(c^2 d+e)e}}{c^4 d^5}$$

$$-\frac{b\sqrt{-\left(c^2 d-2\sqrt{(c^2 d+e)e}+2e\right)d}\; e^2 \operatorname{arctanh}\left(\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)d}{\sqrt{\left(-c^2 d+2\sqrt{(c^2 d+e)e}-2e\right)d}}\right)}{c^2 d^4 (c^2 d+e)}$$

$$-\frac{b\sqrt{-\left(c^2 d-2\sqrt{(c^2 d+e)e}+2e\right)d}\; e^3 \operatorname{arctanh}\left(\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)d}{\sqrt{\left(-c^2 d+2\sqrt{(c^2 d+e)e}-2e\right)d}}\right)}{c^4 d^5 (c^2 d+e)}+\frac{cb\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}}{d^2}$$

$$+\frac{b\sqrt{\left(c^2 d+2\sqrt{(c^2 d+e)e}+2e\right)d}\; e \arctan\left(\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)d}{\sqrt{\left(c^2 d+2\sqrt{(c^2 d+e)e}+2e\right)d}}\right)}{2c^2 d^4}$$

$$+\frac{b\sqrt{\left(c^2 d+2\sqrt{(c^2 d+e)e}+2e\right)d}\; e^2 \arctan\left(\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)d}{\sqrt{\left(c^2 d+2\sqrt{(c^2 d+e)e}+2e\right)d}}\right)}{c^4 d^5}$$

$$+\frac{b\sqrt{-\left(c^2 d-2\sqrt{(c^2 d+e)e}+2e\right)d}\; e \operatorname{arctanh}\left(\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)d}{\sqrt{\left(-c^2 d+2\sqrt{(c^2 d+e)e}-2e\right)d}}\right)}{2c^2 d^4}$$

$$+\frac{b\sqrt{-\left(c^2 d-2\sqrt{(c^2 d+e)e}+2e\right)d}\; e^2 \operatorname{arctanh}\left(\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)d}{\sqrt{\left(-c^2 d+2\sqrt{(c^2 d+e)e}-2e\right)d}}\right)}{c^4 d^5}$$

$$+\frac{b\sqrt{\left(c^2 d+2\sqrt{(c^2 d+e)e}+2e\right)d}\; e \arctan\left(\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)d}{\sqrt{\left(c^2 d+2\sqrt{(c^2 d+e)e}+2e\right)d}}\right)\sqrt{(c^2 d+e)e}}{2c^2 d^4 (c^2 d+e)}$$

$$+\frac{b\sqrt{\left(c^2 d+2\sqrt{(c^2 d+e)\,e}+2e\right)d}\;e^2\arctan\left(\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)d}{\sqrt{\left(c^2 d+2\sqrt{(c^2 d+e)\,e}+2e\right)d}}\right)\sqrt{(c^2 d+e)\,e}}{c^4 d^5\,(c^2 d+e)}$$

$$-\frac{b\sqrt{-\left(c^2 d-2\sqrt{(c^2 d+e)\,e}+2e\right)d}\;e\,\mathrm{arctanh}\left(\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)d}{\sqrt{\left(-c^2 d+2\sqrt{(c^2 d+e)\,e}-2e\right)d}}\right)\sqrt{(c^2 d+e)\,e}}{2\,c^2 d^4\,(c^2 d+e)}$$

$$-\frac{b\sqrt{-\left(c^2 d-2\sqrt{(c^2 d+e)\,e}+2e\right)d}\;e^2\,\mathrm{arctanh}\left(\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)d}{\sqrt{\left(-c^2 d+2\sqrt{(c^2 d+e)\,e}-2e\right)d}}\right)\sqrt{(c^2 d+e)\,e}}{c^4 d^5\,(c^2 d+e)}-\frac{b\,\mathrm{arcsech}(cx)\,e\,c^2 x}{2\,d^2\,(c^2 e x^2+c^2 d)}$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3\,(a+b\,\mathrm{arcsech}(cx))}{(e x^2+d)^3}\,dx$$

Optimal(type 3, 147 leaves, 6 steps):

$$\frac{x^4\,(a+b\,\mathrm{arcsech}(cx))}{4\,d\,(e x^2+d)^2}-\frac{b\,(c^2 d+2e)\,\mathrm{arctanh}\left(\frac{\sqrt{e}\,\sqrt{-c^2x^2+1}}{\sqrt{c^2 d+e}}\right)\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}}{8\,d\,e^{3/2}\,(c^2 d+e)^{3/2}}+\frac{b\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\sqrt{-c^2x^2+1}}{8\,e\,(c^2 d+e)\,(e x^2+d)}$$

Result(type ?, 3330 leaves): Display of huge result suppressed!

Problem 38: Result is not expressed in closed-form.

$$\int \frac{a+b\,\mathrm{arcsech}(cx)}{x\,(e x^2+d)^3}\,dx$$

Optimal(type 4, 833 leaves, 30 steps):

$$\frac{e^2\,(a+b\,\mathrm{arcsech}(cx))}{4\,d^3\left(e+\frac{d}{x^2}\right)^2}-\frac{e\,(a+b\,\mathrm{arcsech}(cx))}{d^3\left(e+\frac{d}{x^2}\right)}+\frac{(a+b\,\mathrm{arcsech}(cx))^2}{2\,b\,d^3}-\frac{(a+b\,\mathrm{arcsech}(cx))\ln\left(1-\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{2\,d^3}$$

$$-\frac{(a+b\,\mathrm{arcsech}(cx))\ln\left(1+\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$

$$-\frac{(a+b\,\mathrm{arcsech}(cx))\ln\left(1-\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

$$-\frac{(a+b\,\mathrm{arcsech}(cx))\ln\left(1+\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}-\frac{b\,\mathrm{polylog}\left(2,-\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}$$

$$-\frac{b\,\mathrm{polylog}\left(2,\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}-\frac{b\,\mathrm{polylog}\left(2,-\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}$$

$$-\frac{b\,\mathrm{polylog}\left(2,\frac{c\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}-\frac{b\,e\left(c^2-\frac{1}{x^2}\right)}{8\,c\,d^2\,(c^2d+e)\left(e+\frac{d}{x^2}\right)x\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}$$

$$-\frac{b\,(c^2d+2e)\,\mathrm{arctanh}\left(\frac{\sqrt{c^2d+e}}{cx\sqrt{e}\,\sqrt{-1+\frac{1}{c^2x^2}}}\right)\sqrt{e}\,\sqrt{-1+\frac{1}{c^2x^2}}}{8d^3\,(c^2d+e)^{3/2}\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}+\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{c^2d+e}}{cx\sqrt{e}\,\sqrt{-1+\frac{1}{c^2x^2}}}\right)\sqrt{e}\,\sqrt{-1+\frac{1}{c^2x^2}}}{d^3\sqrt{c^2d+e}\,\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}$$

Result(type 7, 1548 leaves):

$$-\frac{a\ln(c^2ex^2+c^2d)}{2d^3}+\frac{a\,c^4}{4d\,(c^2ex^2+c^2d)^2}+\frac{a\,c^2}{2d^2\,(c^2ex^2+c^2d)}+\frac{a\ln(cx)}{d^3}-\frac{b\,\mathrm{arcsech}(cx)^2}{2d^3}+\frac{b\,c^5e^2\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,x^3}{8d^2\,(c^2d+e)\,(c^2ex^2+c^2d)^2}$$

$$+\frac{b\,c^5e\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,x}{8d\,(c^2d+e)\,(c^2ex^2+c^2d)^2}-\frac{3b\,c^6e^2\,\mathrm{arcsech}(cx)\,x^4}{4d^2\,(c^2d+e)\,(c^2ex^2+c^2d)^2}-\frac{b\,c^6e\,\mathrm{arcsech}(cx)\,x^2}{d\,(c^2d+e)\,(c^2ex^2+c^2d)^2}-\frac{3b\,e^3\,\mathrm{arcsech}(cx)\,c^4x^4}{4d^3\,(c^2d+e)\,(c^2ex^2+c^2d)^2}$$

$$- \frac{b c^4 e^2 \operatorname{arcsech}(c x) x^2}{d^2 (c^2 d + e) (c^2 e x^2 + c^2 d)^2} - \frac{b e^3 c^4 x^4}{8 d^3 (c^2 d + e) (c^2 e x^2 + c^2 d)^2} - \frac{b c^4 e^2 x^2}{4 d^2 (c^2 d + e) (c^2 e x^2 + c^2 d)^2} - \frac{b c^4 e}{8 d (c^2 d + e) (c^2 e x^2 + c^2 d)^2}$$

$$- \frac{3 b \sqrt{(c^2 d + e) e}\, e \operatorname{arctanh}\left(\frac{2 c^2 d \left(\frac{1}{c x} + \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}} \right)^2 + 2 c^2 d + 4 e}{4 \sqrt{c^2 d e + e^2}} \right)}{4 d^3 (c^2 d + e)^2}$$

$$+ \frac{1}{d^3 (c^2 d + e)} \left(b e^2 \left(\sum_{\_R1 = RootOf(c^2 d \_Z^4 + (2 c^2 d + 4 e) \_Z^2 + c^2 d)} \frac{\operatorname{arcsech}(c x) \ln\left(\frac{\_R1 - \frac{1}{c x} - \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{\_R1} \right) + \operatorname{dilog}\left(\frac{\_R1 - \frac{1}{c x} - \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{\_R1} \right)}{\_R1^2 c^2 d + c^2 d + 2 e} \right) \right)$$

$$+ \frac{b c^2 \operatorname{arcsech}(c x)^2}{d^2 (c^2 d + e)} - \frac{1}{2 d^2 (c^2 d + e)} \left(b c^2 \left(\sum_{\_R1 = RootOf(c^2 d \_Z^4 + (2 c^2 d + 4 e) \_Z^2 + c^2 d)} \frac{(\_R1^2 c^2 d + 2 c^2 d + 4 e) \left(\operatorname{arcsech}(c x) \ln\left(\frac{\_R1 - \frac{1}{c x} - \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{\_R1} \right) + \operatorname{dilog}\left(\frac{\_R1 - \frac{1}{c x} - \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{\_R1} \right) \right)}{\_R1^2 c^2 d + c^2 d + 2 e} \right) \right)$$

$$+ \frac{1}{2 d (c^2 d + e)} \left(b c^4 \left(\sum_{\_R1 = RootOf(c^2 d \_Z^4 + (2 c^2 d + 4 e) \_Z^2 + c^2 d)} \frac{\operatorname{arcsech}(c x) \ln\left(\frac{\_R1 - \frac{1}{c x} - \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{\_R1} \right) + \operatorname{dilog}\left(\frac{\_R1 - \frac{1}{c x} - \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{\_R1} \right)}{\_R1^2 c^2 d + c^2 d + 2 e} \right) \right)$$

$$+\frac{b\,\mathrm{arcsech}(cx)^2 e}{d^3\,(c^2 d+e)}-\frac{1}{2\,d^3\,(c^2 d+e)}\Bigg(b\,e\Bigg(\sum_{\_R1=RootOf(c^2 d\,\_Z^4+(2\,c^2 d+4\,e)\,\_Z^2+c^2 d)}\frac{(\_R1^2 c^2 d+2\,c^2 d+4\,e)\left(\mathrm{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{\_R1}\right)\right)}{\_R1^2 c^2 d+c^2 d+2\,e}\Bigg)\Bigg)$$

$$-\frac{7\,b\,c^2\sqrt{(c^2 d+e)\,e}\;\mathrm{arctanh}\left(\frac{2\,c^2 d\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}\right)^2+2\,c^2 d+4\,e}{4\sqrt{c^2 d e+e^2}}\right)}{8\,d^2\,(c^2 d+e)^2}+\frac{1}{2\,d^2\,(c^2 d+e)}\Bigg(3\,b\,c^2 e\Bigg(\sum_{\_R1=RootOf(c^2 d\,\_Z^4+(2\,c^2 d+4\,e)\,\_Z^2+c^2 d)}\frac{\mathrm{arcsech}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-\frac{1}{cx}-\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{\_R1}\right)}{\_R1^2 c^2 d+c^2 d+2\,e}\Bigg)\Bigg)$$

Problem 39: Unable to integrate problem.

$$\int x^3\,(a+b\,\mathrm{arcsech}(cx))\sqrt{ex^2+d}\,dx$$

Optimal(type 3, 273 leaves, 11 steps):

$$-\frac{d\,(ex^2+d)^{3/2}\,(a+b\,\mathrm{arcsech}(cx))}{3\,e^2}+\frac{(ex^2+d)^{5/2}\,(a+b\,\mathrm{arcsech}(cx))}{5\,e^2}$$

$$+\frac{b\,(15\,c^4 d^2-10\,c^2 d\,e-9\,e^2)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2+1}}{c\sqrt{ex^2+d}}\right)\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}}{120\,c^5 e^{3/2}}+\frac{2\,b\,d^{5/2}\,\mathrm{arctanh}\left(\frac{\sqrt{ex^2+d}}{\sqrt{d}\sqrt{-c^2x^2+1}}\right)\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}}{15\,e^2}$$

$$-\frac{b\,(ex^2+d)^{3/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{-c^2x^2+1}}{20\,c^2 e}-\frac{b\,(c^2 d+9\,e)\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{-c^2x^2+1}\sqrt{ex^2+d}}{120\,c^4 e}$$

Result(type 8, 23 leaves):

$$\int x^3 \left(a + b\,\mathrm{arcsech}(c x)\right) \sqrt{e x^2 + d}\, dx$$

Problem 43: Unable to integrate problem.

$$\int \frac{\left(e x^2 + d\right)^{3/2} \left(a + b\,\mathrm{arcsech}(c x)\right)}{x^6}\, dx$$

Optimal(type 4, 357 leaves, 10 steps):

$$-\frac{\left(e x^2 + d\right)^{5/2} \left(a + b\,\mathrm{arcsech}(c x)\right)}{5 d x^5} + \frac{b \left(e x^2 + d\right)^{3/2} \sqrt{\frac{1}{c x + 1}}\, \sqrt{c x + 1}\, \sqrt{-c^2 x^2 + 1}}{25 x^5} + \frac{4 b \left(c^2 d + 2 e\right) \sqrt{\frac{1}{c x + 1}}\, \sqrt{c x + 1}\, \sqrt{-c^2 x^2 + 1}\, \sqrt{e x^2 + d}}{75 x^3}$$
$$+ \frac{b \left(8 c^4 d^2 + 23 c^2 d e + 23 e^2\right) \sqrt{\frac{1}{c x + 1}}\, \sqrt{c x + 1}\, \sqrt{-c^2 x^2 + 1}\, \sqrt{e x^2 + d}}{75 d x}$$
$$+ \frac{b c \left(8 c^4 d^2 + 23 c^2 d e + 23 e^2\right) \mathrm{EllipticE}\left(c x, \sqrt{-\frac{e}{c^2 d}}\right) \sqrt{\frac{1}{c x + 1}}\, \sqrt{c x + 1}\, \sqrt{e x^2 + d}}{75 d \sqrt{1 + \frac{e x^2}{d}}}$$
$$- \frac{b \left(c^2 d + e\right) \left(8 c^4 d^2 + 19 c^2 d e + 15 e^2\right) \mathrm{EllipticF}\left(c x, \sqrt{-\frac{e}{c^2 d}}\right) \sqrt{\frac{1}{c x + 1}}\, \sqrt{c x + 1}\, \sqrt{1 + \frac{e x^2}{d}}}{75 c d \sqrt{e x^2 + d}}$$

Result(type 8, 23 leaves):

$$\int \frac{\left(e x^2 + d\right)^{3/2} \left(a + b\,\mathrm{arcsech}(c x)\right)}{x^6}\, dx$$

Problem 44: Unable to integrate problem.

$$\int \frac{x \left(a + b\,\mathrm{arcsech}(c x)\right)}{\sqrt{e x^2 + d}}\, dx$$

Optimal(type 3, 127 leaves, 10 steps):

$$-\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{e x^2 + d}}{\sqrt{d}\, \sqrt{-c^2 x^2 + 1}}\right) \sqrt{d}\, \sqrt{\frac{1}{c x + 1}}\, \sqrt{c x + 1}}{e} - \frac{b \arctan\left(\frac{\sqrt{e}\, \sqrt{-c^2 x^2 + 1}}{c \sqrt{e x^2 + d}}\right) \sqrt{\frac{1}{c x + 1}}\, \sqrt{c x + 1}}{c \sqrt{e}} + \frac{\left(a + b\,\mathrm{arcsech}(c x)\right) \sqrt{e x^2 + d}}{e}$$

Result(type 8, 21 leaves):

$$\int \frac{x \left(a + b \operatorname{arcsech}(c x)\right)}{\sqrt{e x^2 + d}} \, dx$$

Problem 48: Unable to integrate problem.

$$\int \frac{a + b \operatorname{arcsech}(c x)}{x^2 \left(e x^2 + d\right)^{3/2}} \, dx$$

Optimal(type 4, 225 leaves, 8 steps):

$$\frac{-a - b \operatorname{arcsech}(c x)}{d x \sqrt{e x^2 + d}} - \frac{2 e x \left(a + b \operatorname{arcsech}(c x)\right)}{d^2 \sqrt{e x^2 + d}} + \frac{b \sqrt{\frac{1}{c x + 1}} \sqrt{c x + 1} \sqrt{-c^2 x^2 + 1} \sqrt{e x^2 + d}}{d^2 x}$$
$$+ \frac{b c \operatorname{EllipticE}\left(c x, \sqrt{-\frac{e}{c^2 d}}\right) \sqrt{\frac{1}{c x + 1}} \sqrt{c x + 1} \sqrt{e x^2 + d}}{d^2 \sqrt{1 + \frac{e x^2}{d}}} - \frac{b \left(c^2 d + 2 e\right) \operatorname{EllipticF}\left(c x, \sqrt{-\frac{e}{c^2 d}}\right) \sqrt{\frac{1}{c x + 1}} \sqrt{c x + 1} \sqrt{1 + \frac{e x^2}{d}}}{c d^2 \sqrt{e x^2 + d}}$$

Result(type 8, 23 leaves):

$$\int \frac{a + b \operatorname{arcsech}(c x)}{x^2 \left(e x^2 + d\right)^{3/2}} \, dx$$

Problem 49: Unable to integrate problem.

$$\int \frac{x^3 \left(a + b \operatorname{arcsech}(c x)\right)}{\sqrt{-c^4 x^4 + 1}} \, dx$$

Optimal(type 3, 135 leaves, 7 steps):

$$\frac{b \operatorname{arctanh}\left(\sqrt{c^2 x^2 + 1}\right) \sqrt{-c^2 x^2 + 1}}{2 c^5 x \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} - \frac{b \sqrt{-c^2 x^2 + 1} \sqrt{c^2 x^2 + 1}}{2 c^5 x \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} - \frac{\left(a + b \operatorname{arcsech}(c x)\right) \sqrt{-c^4 x^4 + 1}}{2 c^4}$$

Result(type 8, 26 leaves):

$$\int \frac{x^3 \left(a + b \operatorname{arcsech}(c x)\right)}{\sqrt{-c^4 x^4 + 1}} \, dx$$

Test results for the 28 problems in "7.5.2 Inverse hyperbolic secant functions.txt"

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{arcsech}(x b + a)}{x^3}\, dx$$

Optimal(type 3, 117 leaves, 7 steps):

$$\frac{b^2 \operatorname{arcsech}(x b + a)}{2 a^2} - \frac{\operatorname{arcsech}(x b + a)}{2 x^2} - \frac{(-2 a^2 + 1) b^2 \operatorname{arctanh}\left(\frac{\sqrt{1 + a} \tanh\left(\frac{\operatorname{arcsech}(x b + a)}{2}\right)}{\sqrt{1 - a}}\right)}{a^2 (-a^2 + 1)^{3/2}} + \frac{b (x b + a + 1) \sqrt{\frac{-x b - a + 1}{x b + a + 1}}}{2 a (-a^2 + 1) x}$$

Result(type 3, 878 leaves):

$$- \frac{\operatorname{arcsech}(x b + a)}{2 x^2} + \frac{b^3 \sqrt{-\frac{x b + a - 1}{x b + a}} \sqrt{\frac{x b + a + 1}{x b + a}}\, x \operatorname{arctanh}\left(\frac{1}{\sqrt{-(x b + a)^2 + 1}}\right)}{2 \sqrt{-(x b + a)^2 + 1}\, (-1 + a) (1 + a)}$$

$$+ \frac{b^2 \sqrt{-\frac{x b + a - 1}{x b + a}} \sqrt{\frac{x b + a + 1}{x b + a}}\, a \operatorname{arctanh}\left(\frac{1}{\sqrt{-(x b + a)^2 + 1}}\right)}{2 \sqrt{-(x b + a)^2 + 1}\, (-1 + a) (1 + a)}$$

$$- \frac{b^3 \sqrt{-\frac{x b + a - 1}{x b + a}} \sqrt{\frac{x b + a + 1}{x b + a}}\, x \ln\left(\frac{2 \left(\sqrt{-a^2 + 1} \sqrt{-(x b + a)^2 + 1} - a (x b + a) + 1\right)}{b x}\right)}{\sqrt{-(x b + a)^2 + 1}\, (-1 + a) (1 + a) \sqrt{-a^2 + 1}}$$

$$- \frac{b^3 \sqrt{-\frac{x b + a - 1}{x b + a}} \sqrt{\frac{x b + a + 1}{x b + a}}\, x \operatorname{arctanh}\left(\frac{1}{\sqrt{-(x b + a)^2 + 1}}\right)}{2 \sqrt{-(x b + a)^2 + 1}\, a^2 (-1 + a) (1 + a)}$$

$$- \frac{b^2 \sqrt{-\frac{x b + a - 1}{x b + a}} \sqrt{\frac{x b + a + 1}{x b + a}}\, a \ln\left(\frac{2 \left(\sqrt{-a^2 + 1} \sqrt{-(x b + a)^2 + 1} - a (x b + a) + 1\right)}{b x}\right)}{\sqrt{-(x b + a)^2 + 1}\, (-1 + a) (1 + a) \sqrt{-a^2 + 1}}$$

$$- \frac{b^2 \sqrt{-\frac{x b + a - 1}{x b + a}} \sqrt{\frac{x b + a + 1}{x b + a}} \operatorname{arctanh}\left(\frac{1}{\sqrt{-(x b + a)^2 + 1}}\right)}{2 \sqrt{-(x b + a)^2 + 1}\, a (-1 + a) (1 + a)}$$

$$+ \frac{b^3 \sqrt{-\frac{x b + a - 1}{x b + a}} \sqrt{\frac{x b + a + 1}{x b + a}}\, x \ln\left(\frac{2 \left(\sqrt{-a^2 + 1} \sqrt{-(x b + a)^2 + 1} - a (x b + a) + 1\right)}{b x}\right)}{2 \sqrt{-(x b + a)^2 + 1}\, a^2 (-1 + a) (1 + a) \sqrt{-a^2 + 1}} - \frac{b^2 \sqrt{-\frac{x b + a - 1}{x b + a}} \sqrt{\frac{x b + a + 1}{x b + a}}}{2 a (-1 + a) (1 + a)}$$

$$+ \frac{b^2 \sqrt{-\frac{x b + a - 1}{x b + a}} \sqrt{\frac{x b + a + 1}{x b + a}} \ln\left(\frac{2 \left(\sqrt{-a^2 + 1} \sqrt{-(x b + a)^2 + 1} - a (x b + a) + 1\right)}{b x}\right)}{2 \sqrt{-(x b + a)^2 + 1}\, a (-1 + a) (1 + a) \sqrt{-a^2 + 1}} - \frac{b \sqrt{-\frac{x b + a - 1}{x b + a}} \sqrt{\frac{x b + a + 1}{x b + a}}}{2 (-1 + a) (1 + a) x}$$

Problem 5: Unable to integrate problem.

$$\int x \operatorname{arcsech}(x b + a)^3 \, dx$$

Optimal(type 4, 401 leaves, 16 steps):

$$-\frac{3 \operatorname{arcsech}(x b + a)^2}{2 b^2} - \frac{a^2 \operatorname{arcsech}(x b + a)^3}{2 b^2} + \frac{x^2 \operatorname{arcsech}(x b + a)^3}{2} + \frac{6 a \operatorname{arcsech}(x b + a)^2 \arctan\left(\frac{1}{x b + a} + \sqrt{\frac{1}{x b + a} - 1}\, \sqrt{\frac{1}{x b + a} + 1}\right)}{b^2}$$
$$+ \frac{3 \operatorname{arcsech}(x b + a) \ln\left(1 + \left(\frac{1}{x b + a} + \sqrt{\frac{1}{x b + a} - 1}\, \sqrt{\frac{1}{x b + a} + 1}\right)^2\right)}{b^2}$$
$$- \frac{6 I a \operatorname{arcsech}(x b + a) \operatorname{polylog}\left(2, -I \left(\frac{1}{x b + a} + \sqrt{\frac{1}{x b + a} - 1}\, \sqrt{\frac{1}{x b + a} + 1}\right)\right)}{b^2}$$
$$+ \frac{6 I a \operatorname{arcsech}(x b + a) \operatorname{polylog}\left(2, I \left(\frac{1}{x b + a} + \sqrt{\frac{1}{x b + a} - 1}\, \sqrt{\frac{1}{x b + a} + 1}\right)\right)}{b^2} + \frac{3 \operatorname{polylog}\left(2, -\left(\frac{1}{x b + a} + \sqrt{\frac{1}{x b + a} - 1}\, \sqrt{\frac{1}{x b + a} + 1}\right)^2\right)}{2 b^2}$$
$$+ \frac{6 I a \operatorname{polylog}\left(3, -I \left(\frac{1}{x b + a} + \sqrt{\frac{1}{x b + a} - 1}\, \sqrt{\frac{1}{x b + a} + 1}\right)\right)}{b^2} - \frac{6 I a \operatorname{polylog}\left(3, I \left(\frac{1}{x b + a} + \sqrt{\frac{1}{x b + a} - 1}\, \sqrt{\frac{1}{x b + a} + 1}\right)\right)}{b^2}$$
$$- \frac{3 (x b + a + 1) \operatorname{arcsech}(x b + a)^2 \sqrt{\frac{-x b - a + 1}{x b + a + 1}}}{2 b^2}$$

Result(type 8, 12 leaves):

$$\int x \operatorname{arcsech}(x b + a)^3 \, dx$$

Problem 6: Unable to integrate problem.

$$\int \operatorname{arcsech}(x b + a)^3 \, dx$$

Optimal(type 4, 243 leaves, 10 steps):

$$\frac{(x b + a) \operatorname{arcsech}(x b + a)^3}{b} - \frac{6 \operatorname{arcsech}(x b + a)^2 \arctan\left(\frac{1}{x b + a} + \sqrt{\frac{1}{x b + a} - 1}\, \sqrt{\frac{1}{x b + a} + 1}\right)}{b}$$
$$+ \frac{6 I \operatorname{arcsech}(x b + a) \operatorname{polylog}\left(2, -I \left(\frac{1}{x b + a} + \sqrt{\frac{1}{x b + a} - 1}\, \sqrt{\frac{1}{x b + a} + 1}\right)\right)}{b}$$

$$-\frac{6\,\mathrm{I}\,\mathrm{arcsech}(x b+a)\,\mathrm{polylog}\left(2,\mathrm{I}\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)\right)}{b}-\frac{6\,\mathrm{I}\,\mathrm{polylog}\left(3,-\mathrm{I}\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)\right)}{b}$$

$$+\frac{6\,\mathrm{I}\,\mathrm{polylog}\left(3,\mathrm{I}\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)\right)}{b}$$

Result(type 8, 10 leaves):

$$\int \mathrm{arcsech}(x b+a)^3\,\mathrm{d}x$$

Problem 7: Unable to integrate problem.

$$\int \frac{\mathrm{arcsech}(x b+a)^3}{x}\,\mathrm{d}x$$

Optimal(type 4, 632 leaves, 20 steps):

$$-\mathrm{arcsech}(x b+a)^3\ln\left(1+\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)^2\right)+\mathrm{arcsech}(x b+a)^3\ln\left(1-\frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)}{1-\sqrt{-a^2+1}}\right)$$

$$+\mathrm{arcsech}(x b+a)^3\ln\left(1-\frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)}{1+\sqrt{-a^2+1}}\right)$$

$$-\frac{3\,\mathrm{arcsech}(x b+a)^2\,\mathrm{polylog}\left(2,-\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)^2\right)}{2}+3\,\mathrm{arcsech}(x b+a)^2\,\mathrm{polylog}\left(2,\right.$$

$$\left.\frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)}{1-\sqrt{-a^2+1}}\right)+3\,\mathrm{arcsech}(x b+a)^2\,\mathrm{polylog}\left(2,\frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)}{1+\sqrt{-a^2+1}}\right)$$

$$+\frac{3\,\mathrm{arcsech}(x b+a)\,\mathrm{polylog}\left(3,-\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)^2\right)}{2}-6\,\mathrm{arcsech}(x b+a)\,\mathrm{polylog}\left(3,\right.$$

$$\left.\frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)}{1-\sqrt{-a^2+1}}\right)-6\,\mathrm{arcsech}(x b+a)\,\mathrm{polylog}\left(3,\frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)}{1+\sqrt{-a^2+1}}\right)$$

$$-\frac{3\,\mathrm{polylog}\left(4, -\left(\frac{1}{xb+a}+\sqrt{\frac{1}{xb+a}-1}\,\sqrt{\frac{1}{xb+a}+1}\right)^{2}\right)}{4}+6\,\mathrm{polylog}\left(4, \frac{a\left(\frac{1}{xb+a}+\sqrt{\frac{1}{xb+a}-1}\,\sqrt{\frac{1}{xb+a}+1}\right)}{1-\sqrt{-a^{2}+1}}\right)+6\,\mathrm{polylog}\left(4, \frac{a\left(\frac{1}{xb+a}+\sqrt{\frac{1}{xb+a}-1}\,\sqrt{\frac{1}{xb+a}+1}\right)}{1+\sqrt{-a^{2}+1}}\right)$$

Result(type 8, 14 leaves):

$$\int \frac{\mathrm{arcsech}(xb+a)^{3}}{x}\, dx$$

Problem 8: Unable to integrate problem.

$$\int \frac{\mathrm{arcsech}(xb+a)^{3}}{x^{2}}\, dx$$

Optimal(type 4, 444 leaves, 14 steps):

$$-\frac{b\,\mathrm{arcsech}(xb+a)^{3}}{a}-\frac{\mathrm{arcsech}(xb+a)^{3}}{x}+\frac{3\,b\,\mathrm{arcsech}(xb+a)^{2}\ln\left(1-\frac{a\left(\frac{1}{xb+a}+\sqrt{\frac{1}{xb+a}-1}\,\sqrt{\frac{1}{xb+a}+1}\right)}{1-\sqrt{-a^{2}+1}}\right)}{a\sqrt{-a^{2}+1}}$$

$$-\frac{3\,b\,\mathrm{arcsech}(xb+a)^{2}\ln\left(1-\frac{a\left(\frac{1}{xb+a}+\sqrt{\frac{1}{xb+a}-1}\,\sqrt{\frac{1}{xb+a}+1}\right)}{1+\sqrt{-a^{2}+1}}\right)}{a\sqrt{-a^{2}+1}}$$

$$+\frac{6\,b\,\mathrm{arcsech}(xb+a)\,\mathrm{polylog}\left(2, \frac{a\left(\frac{1}{xb+a}+\sqrt{\frac{1}{xb+a}-1}\,\sqrt{\frac{1}{xb+a}+1}\right)}{1-\sqrt{-a^{2}+1}}\right)}{a\sqrt{-a^{2}+1}}$$

$$-\frac{6\,b\,\mathrm{arcsech}(xb+a)\,\mathrm{polylog}\left(2, \frac{a\left(\frac{1}{xb+a}+\sqrt{\frac{1}{xb+a}-1}\,\sqrt{\frac{1}{xb+a}+1}\right)}{1+\sqrt{-a^{2}+1}}\right)}{a\sqrt{-a^{2}+1}}$$

$$-\frac{6 b \operatorname{polylog}\left(3, \frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)}{1-\sqrt{-a^2+1}}\right)}{a\sqrt{-a^2+1}}+\frac{6 b \operatorname{polylog}\left(3, \frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)}{1+\sqrt{-a^2+1}}\right)}{a\sqrt{-a^2+1}}$$

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Result(type 8, 14 leaves):
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$$\int \frac{\operatorname{arcsech}(x b+a)^3}{x^2}\, dx$$

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Problem 9: Unable to integrate problem.
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$$\int \frac{\operatorname{arcsech}(x b+a)^3}{x^3}\, dx$$

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Optimal(type 4, 1263 leaves, 32 steps):
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$$-\frac{3 b^2 \operatorname{arcsech}(x b+a)^2}{2 a^2\left(-a^2+1\right)}+\frac{b^2 \operatorname{arcsech}(x b+a)^3}{2 a^2}-\frac{\operatorname{arcsech}(x b+a)^3}{2 x^2}+\frac{3 b^2 \operatorname{arcsech}(x b+a) \ln\left(1-\frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)}{1-\sqrt{-a^2+1}}\right)}{a^2\left(-a^2+1\right)}$$

$$+\frac{3 b^2 \operatorname{arcsech}(x b+a)^2 \ln\left(1-\frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)}{1-\sqrt{-a^2+1}}\right)}{2 a^2\left(-a^2+1\right)^{3/2}}$$

$$+\frac{3 b^2 \operatorname{arcsech}(x b+a) \ln\left(1-\frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)}{1+\sqrt{-a^2+1}}\right)}{a^2\left(-a^2+1\right)}$$

$$-\frac{3 b^2 \operatorname{arcsech}(x b+a)^2 \ln\left(1-\frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)}{1+\sqrt{-a^2+1}}\right)}{2 a^2\left(-a^2+1\right)^{3/2}}$$

$$+\frac{3 b^2 \operatorname{polylog}\left(2, \frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1}\,\sqrt{\frac{1}{x b+a}+1}\right)}{1-\sqrt{-a^2+1}}\right)}{a^2\left(-a^2+1\right)}$$

$$+\frac{3 b^2 \operatorname{arcsech}(x b+a) \operatorname{polylog}\left(2, \frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1} \sqrt{\frac{1}{x b+a}+1}\right)}{1-\sqrt{-a^2+1}}\right)}{a^2\left(-a^2+1\right)^{3/2}}$$

$$+\frac{3 b^2 \operatorname{polylog}\left(2, \frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1} \sqrt{\frac{1}{x b+a}+1}\right)}{1+\sqrt{-a^2+1}}\right)}{a^2\left(-a^2+1\right)}$$

$$-\frac{3 b^2 \operatorname{arcsech}(x b+a) \operatorname{polylog}\left(2, \frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1} \sqrt{\frac{1}{x b+a}+1}\right)}{1+\sqrt{-a^2+1}}\right)}{a^2\left(-a^2+1\right)^{3/2}}$$

$$-\frac{3 b^2 \operatorname{polylog}\left(3, \frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1} \sqrt{\frac{1}{x b+a}+1}\right)}{1-\sqrt{-a^2+1}}\right)}{a^2\left(-a^2+1\right)^{3/2}}+\frac{3 b^2 \operatorname{polylog}\left(3, \frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1} \sqrt{\frac{1}{x b+a}+1}\right)}{1+\sqrt{-a^2+1}}\right)}{a^2\left(-a^2+1\right)^{3/2}}$$

$$-\frac{3 b^2 \operatorname{arcsech}(x b+a)^2 \ln\left(1-\frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1} \sqrt{\frac{1}{x b+a}+1}\right)}{1-\sqrt{-a^2+1}}\right)}{a^2 \sqrt{-a^2+1}}$$

$$+\frac{3 b^2 \operatorname{arcsech}(x b+a)^2 \ln\left(1-\frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1} \sqrt{\frac{1}{x b+a}+1}\right)}{1+\sqrt{-a^2+1}}\right)}{a^2 \sqrt{-a^2+1}}$$

$$-\frac{6 b^2 \operatorname{arcsech}(x b+a) \operatorname{polylog}\left(2, \frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1} \sqrt{\frac{1}{x b+a}+1}\right)}{1-\sqrt{-a^2+1}}\right)}{a^2 \sqrt{-a^2+1}}$$

$$+\frac{6 b^2 \operatorname{arcsech}(x b+a) \operatorname{polylog}\left(2, \frac{a\left(\frac{1}{x b+a}+\sqrt{\frac{1}{x b+a}-1} \sqrt{\frac{1}{x b+a}+1}\right)}{1+\sqrt{-a^2+1}}\right)}{a^2 \sqrt{-a^2+1}}$$

$$+\frac{6 b^2 \operatorname{polylog}\left(3, \frac{a\left(\frac{1}{xb+a}+\sqrt{\frac{1}{xb+a}-1}\,\sqrt{\frac{1}{xb+a}+1}\right)}{1-\sqrt{-a^2+1}}\right)}{a^2\sqrt{-a^2+1}} - \frac{6 b^2 \operatorname{polylog}\left(3, \frac{a\left(\frac{1}{xb+a}+\sqrt{\frac{1}{xb+a}-1}\,\sqrt{\frac{1}{xb+a}+1}\right)}{1+\sqrt{-a^2+1}}\right)}{a^2\sqrt{-a^2+1}}$$

$$+\frac{3 b^2 (xb+a+1)\operatorname{arcsech}(xb+a)^2\sqrt{\frac{-xb-a+1}{xb+a+1}}}{2a\left(-a^2+1\right)(xb+a)\left(1-\frac{a}{xb+a}\right)}$$

Result(type 8, 14 leaves):

$$\int \frac{\operatorname{arcsech}(xb+a)^3}{x^3}\,dx$$

Problem 18: Unable to integrate problem.

$$\int\left(\frac{1}{a x^3}+\sqrt{\frac{1}{a x^3}-1}\,\sqrt{\frac{1}{a x^3}+1}\right)x^m\,dx$$

Optimal(type 5, 124 leaves, 4 steps):

$$-\frac{3x^{-2+m}}{a\left(-m^2+m+2\right)}+\frac{\left(\frac{1}{a x^3}+\sqrt{\frac{1}{a x^3}-1}\,\sqrt{\frac{1}{a x^3}+1}\right)x^{m+1}}{m+1}-\frac{3x^{-2+m}\operatorname{hypergeom}\left(\left[\frac{1}{2},-\frac{1}{3}+\frac{m}{6}\right],\left[\frac{2}{3}+\frac{m}{6}\right],a^2x^6\right)\sqrt{\frac{1}{a x^3+1}}\,\sqrt{a x^3+1}}{a\left(-m^2+m+2\right)}$$

Result(type 8, 37 leaves):

$$\int\left(\frac{1}{a x^3}+\sqrt{\frac{1}{a x^3}-1}\,\sqrt{\frac{1}{a x^3}+1}\right)x^m\,dx$$

Problem 19: Unable to integrate problem.

$$\int\left(\frac{x}{a}+\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}\right)x^m\,dx$$

Optimal(type 5, 116 leaves, 5 steps):

$$\frac{\left(\frac{x}{a}+\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}\right)x^{m+1}}{m+1}-\frac{x^{2+m}}{a\left(m^2+3m+2\right)}-\frac{x^{2+m}\operatorname{hypergeom}\left(\left[\frac{1}{2},-1-\frac{m}{2}\right],\left[-\frac{m}{2}\right],\frac{a^2}{x^2}\right)\sqrt{\frac{1}{1+\frac{a}{x}}}\,\sqrt{1+\frac{a}{x}}}{a\left(m^2+3m+2\right)}$$

Result(type 8, 31 leaves):

$$\int\left(\frac{x}{a}+\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}\right)x^m\,dx$$

Problem 20: Unable to integrate problem.

$$\int \frac{\frac{1}{a x^{p}}+\sqrt{\frac{1}{a x^{p}}-1}\,\sqrt{\frac{1}{a x^{p}}+1}}{x^{2}}\, dx$$

Optimal(type 5, 128 leaves, 4 steps):

$$-\frac{\frac{1}{a x^{p}}+\sqrt{\frac{1}{a x^{p}}-1}\,\sqrt{\frac{1}{a x^{p}}+1}}{x}+\frac{p x^{-1-p}}{a(1+p)}+\frac{p x^{-1-p}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{-1-p}{2p}\right],\left[\frac{-1+p}{2p}\right],a^{2}x^{2p}\right)\sqrt{\frac{1}{1+a x^{p}}}\,\sqrt{1+a x^{p}}}{a(1+p)}$$

Result(type 8, 43 leaves):

$$\int \frac{\frac{1}{a x^{p}}+\sqrt{\frac{1}{a x^{p}}-1}\,\sqrt{\frac{1}{a x^{p}}+1}}{x^{2}}\, dx$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\frac{1}{a x}+\sqrt{\frac{1}{a x}-1}\,\sqrt{\frac{1}{a x}+1}}\, dx$$

Optimal(type 3, 61 leaves, 6 steps):

$$\frac{\ln(a x+1)}{a}+\frac{2\ln\left(1+\sqrt{\frac{-a x+1}{a x+1}}\right)}{a}-\frac{(a x+1)\sqrt{\frac{-a x+1}{a x+1}}}{a}$$

Result(type ?, 2615 leaves): Display of huge result suppressed!

Problem 25: Unable to integrate problem.

$$\int \frac{1}{\left(\frac{1}{a x}+\sqrt{\frac{1}{a x}-1}\,\sqrt{\frac{1}{a x}+1}\right)x}\, dx$$

Optimal(type 3, 42 leaves, 5 steps):

$$-2\arctan\left(\sqrt{\frac{-a x+1}{a x+1}}\right)-\frac{2}{1+\sqrt{\frac{-a x+1}{a x+1}}}$$

Result(type 8, 39 leaves):

$$\int \frac{1}{\left(\frac{1}{a x}+\sqrt{\frac{1}{a x}-1}\,\sqrt{\frac{1}{a x}+1}\right) x}\,\mathrm{d}x$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(\frac{1}{c x}+\sqrt{-1+\frac{1}{c x}}\,\sqrt{1+\frac{1}{c x}}\right) x}{-c^2 x^2+1}\,\mathrm{d}x$$

Optimal(type 3, 33 leaves, 5 steps):

$$\frac{\operatorname{arctanh}(c x)}{c^2}+\frac{\arcsin(c x)\sqrt{\frac{1}{c x+1}}\,\sqrt{c x+1}}{c^2}$$

Result(type 3, 89 leaves):

$$\frac{\sqrt{-\frac{c x-1}{c x}}\,x\sqrt{\frac{c x+1}{c x}}\,\operatorname{csgn}(c)\arctan\left(\frac{\operatorname{csgn}(c)\,c x}{\sqrt{-c^2 x^2+1}}\right)}{\sqrt{-c^2 x^2+1}\,c}-\frac{\ln(c x-1)}{2 c^2}+\frac{\ln(c x+1)}{2 c^2}$$

Problem 28: Unable to integrate problem.

$$\int x^{-1+n}\operatorname{arcsech}(a+b x^n)\,\mathrm{d}x$$

Optimal(type 3, 56 leaves, 5 steps):

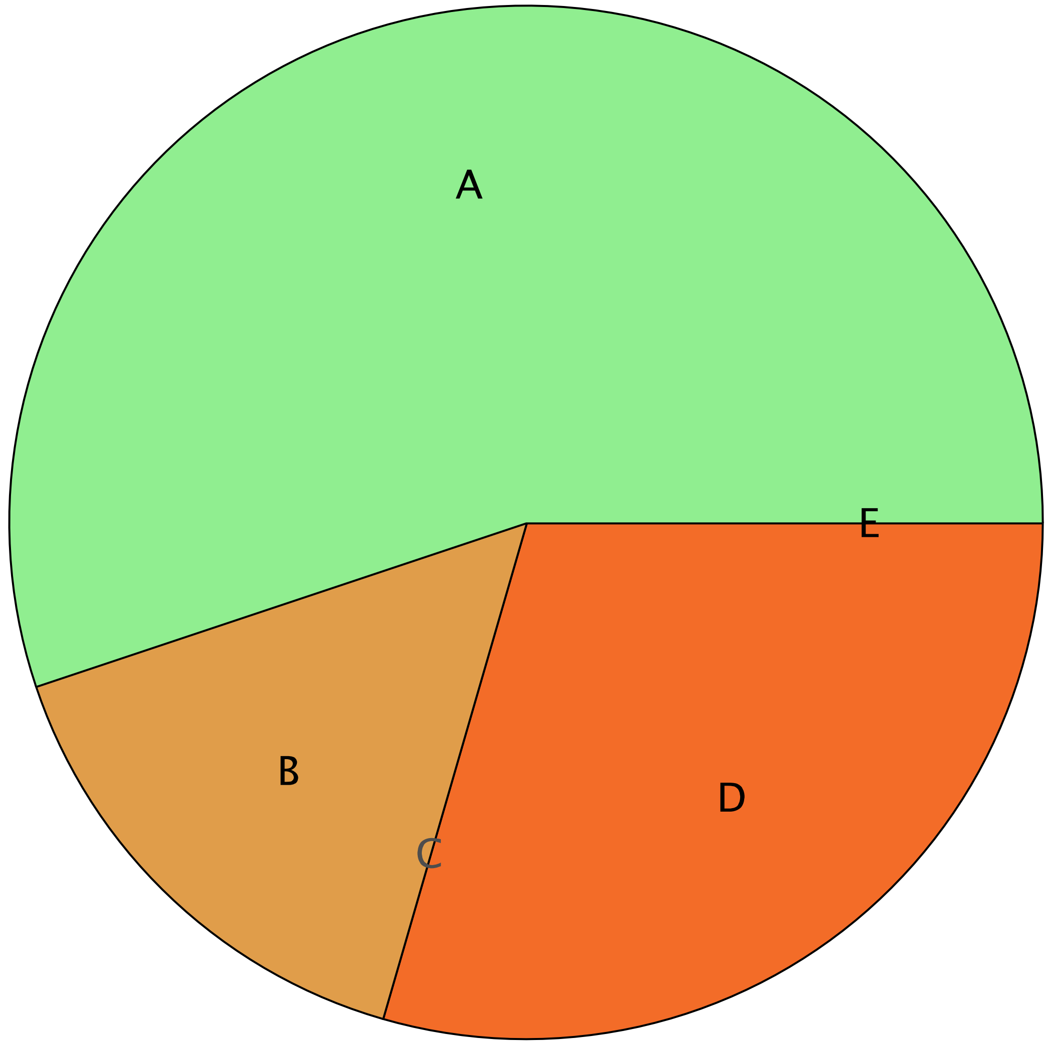
$$\frac{(a+b x^n)\operatorname{arcsech}(a+b x^n)}{b n}-\frac{2\arctan\left(\sqrt{\frac{1-a-b x^n}{1+a+b x^n}}\right)}{b n}$$

Result(type 8, 16 leaves):

$$\int x^{-1+n}\operatorname{arcsech}(a+b x^n)\,\mathrm{d}x$$

Summary of Integration Test Results

78 integration problems



A - 43 optimal antiderivatives
B - 12 more than twice size of optimal antiderivatives
C - 0 unnecessarily complex antiderivatives
D - 23 unable to integrate problems
E - 0 integration timeouts